Math 60 8.5 Linear Functions and Models (2 days)

Objectives: 1) Graph linear functions and vertical lines

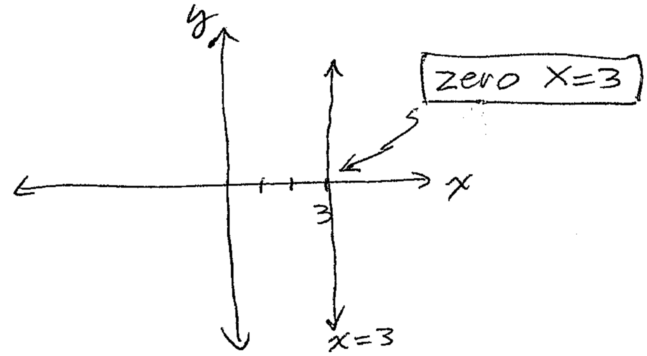
2) Find the zero of a linear function

3) Write the equation of a linear function
- given two points on the line
- given slope and one point

Graph each equation or function and find its zero.

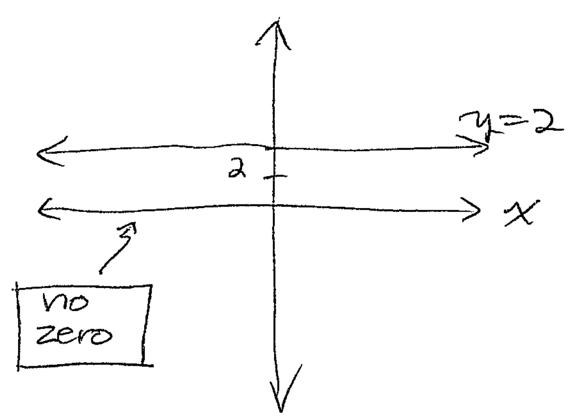
① $x=3$

$x=3$ has no y-coordinate.
All x-coordinates, for every point on the graph, are 3.
vertical line



* Note: We cannot use the $f(x)$ notation because vertical lines are not functions
  1) They fail the Vertical Line Test (VLT)
  2) The x-coordinate 3 has more than one y-coordinate.

② $f(x)=2$
This function has the same graph as $y=2$.
All y coordinates are 2.
horizontal line



* This is a function.
  1) Passes V.L.T.
  2) Every x coordinate has only one y-coord.

Math 60 8.5 Graph and find zero, continued.

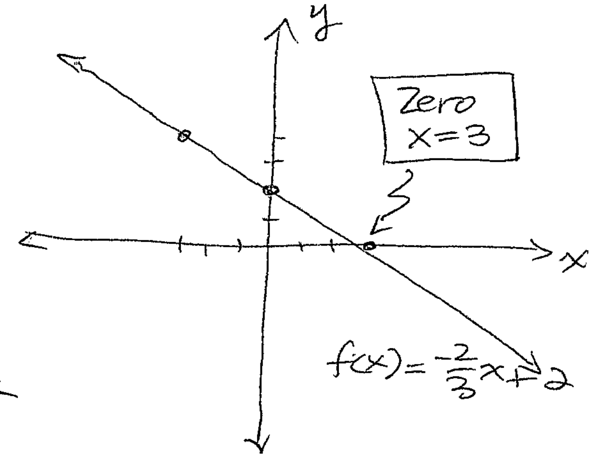
(3) $f(x) = \frac{-2}{3}x + 2$

A linear function is in the form $f(x) = mx + b$, where m is the slope and $(0, b)$ is the y-intercept.

$m = \frac{-2}{3}$

$b = 2 \Rightarrow (0, 2)$

plot y-intercept first
then use $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{3} = \frac{2}{-3}$



If the graph is done neatly, we find the zero, or x-intercept on the grid.
If graph is not neat enough to be reliable, set $y=0$, meaning replace $f(x)$ by 0.

$f(x) = \frac{-2}{3}x + 2$ solve for x

$0 = -\frac{2}{3}x + 2$ subtract 2 both sides

$-2 = \frac{-2}{3}x$ multiply by reciprocal

$\frac{-3}{2} \cdot -2 = \frac{-3}{2} \cdot \frac{-2}{3}x$

$\boxed{3 = x}$

* Memorize *
Slope intercept form
$y = mx + b$
$f(x) = mx + b$

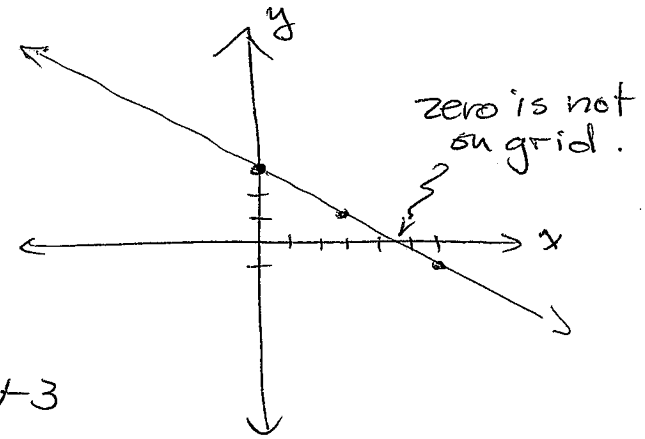
Math 60 8.5 Graph and find zeros continued.

(4) $f(x) = -\frac{2}{3}x + 3$

$y = -\frac{2}{3}x + 3$ replace f(x) by y

$m = -\frac{2}{3}$, $b =$ (0,3)

Set y=0 by replacing f(x) by 0:



$0 = -\frac{2}{3}x + 3$

$-3 = -\frac{2}{3}x$ subtract 3

$-3 \cdot -\frac{3}{2} = -\frac{3}{2} \cdot -\frac{2}{3} x$

$\boxed{\frac{9}{2} = x}$

(5) $f(x) = -\frac{2}{3}x + \frac{4}{3}$

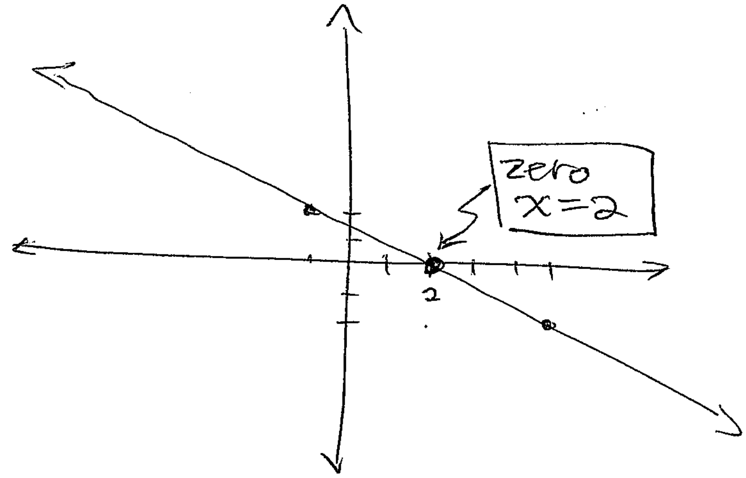
y-intercept b is <u>not</u> an integer, so it's not on the grid!
Try using the zero, or x-intercept, as starting point on the graph.

$0 = -\frac{2}{3}x + \frac{4}{3}$

$-\frac{4}{3} = -\frac{2}{3}x$

$-\frac{4}{3} \cdot -\frac{3}{2} = x$

$\boxed{2 = x}$



Math 60 8.5 Graph and find zero, continued.

(6) $f(x) = -\frac{2}{3}x + \frac{7}{3}$

y intercept $b = \frac{7}{3}$ is not an integer.

Try using the zero

$$0 = \frac{-2}{3}x + \frac{7}{3}$$

$$-\frac{7}{3} = \frac{-2}{3}x$$

$$-\frac{7}{3} \cdot \frac{-3}{2} = x$$

$$\boxed{\frac{7}{2} = x}$$

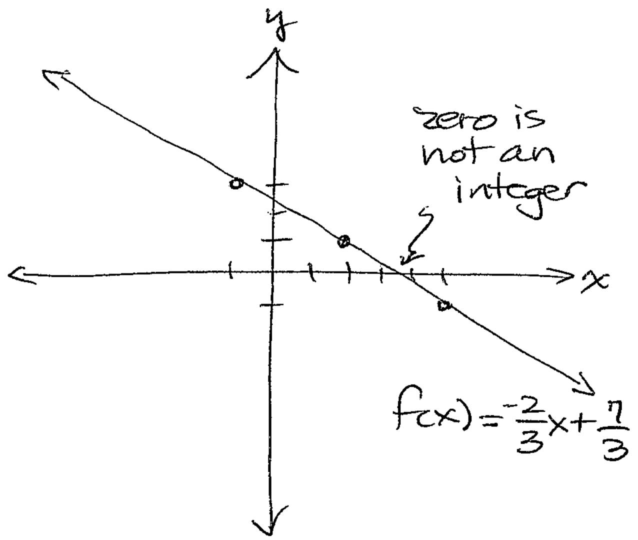
The zero is not an integer either!

Find some other ordered pair, any value, so long as both the x- and y-coordinates are integers (not fractions)

x	y	
0	7/3	no
1	5/3	no
2	1	yes!

Plot (2, 1)

use $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{3} = \frac{2}{-3}$



Math 60 Summary of Techniques for Graphing a Line

Begin with Option 1. If it does not apply, try Option 2. If not Option 3, go on to Option 4, and so on.

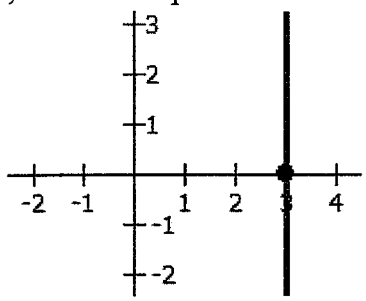
$\boxed{\text{Option 1}}$:

Ask: Is the line vertical? [Does the equation have x but no y?]

Method: Plot x-intercept and a line up and down from it.

Example 1: $x = 3$.

Plot the x-intercept at the value given, (3,0) in this example, and a line up and down from it.



Graph for Example 1

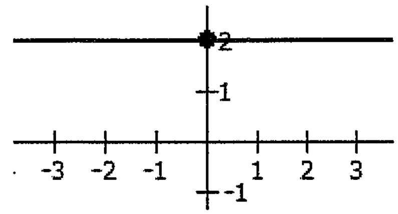
$\boxed{\text{Option 2}}$:

Ask: Is the line horizontal? [Does the equation have y but no x?]

Method: Plot the y-intercept and a line left and right from it.

Example 2: $f(x) = 2$

Plot the y-intercept at the value given, (0,2) in example, and a line left and right from it.



Graph for Example 2

$\boxed{\text{Option 3}}$:

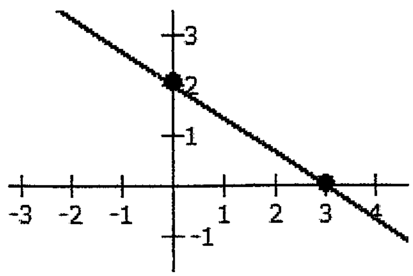
Ask: Are the x-intercept and y-intercept both integers? [Is the constant is evenly divisible by both the coefficient of the x-term and evenly divisible by the coefficient of the y-term?]

Method: Find and plot the x-intercept, find and plot the y-intercept, connect the two with a line.

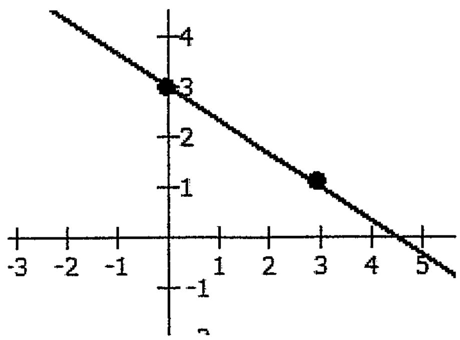
Example 3: $2x + 3y = 6$ [6 is divisible by 2 and divisible by 3]

Find the x-intercept (set y=0, solve for x), (3,0) in example, and plot it.

Find the y-intercept (set x=0, solve for y), (0,2) in example, and plot it. Connect with a line.



Graph for Example 3



Graph for Example 4

$\boxed{\text{Option 4}}$:

Ask: Is the y-intercept an integer? [Is the constant term is divisible by the y-coefficient?]

Method: Write equation in slope-intercept ($f(x) = mx + b$) form, plot the y-intercept, use the slope.

Example 4: $2x + 3y = 9$ [9 is divisible by y-coefficient 3, but not by x-coefficient 2]

Write in slope-intercept form: $f(x) = -\frac{2}{3}x + 3$.

Continued...

Plot the y-intercept, (0,3) in example. (continued on the back)

Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example).

From the y-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

Option 5:

Ask: Is the x-intercept an integer? [Is the constant term is divisible by the x-coefficient?]

Method: Write equation in slope-intercept ($y = mx + b$) form, find and plot the x-intercept, use the slope.

Example 5: $2x + 3y = 4$ [4 is divisible by x-coefficient 2 but not by y-coefficient 3]

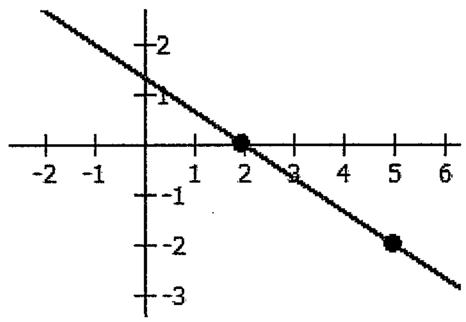
Write in slope-intercept form: $f(x) = -\frac{2}{3}x + \frac{4}{3}$

Find and plot x-intercept (2,0).

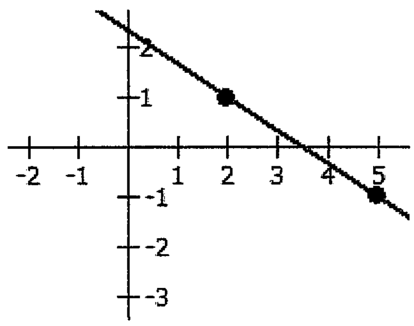
Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)

From the x-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).



Graph for Example 5



Graph for Example 6

Option 6:

Ask: Is neither the x-intercept nor y-intercept an integer? [Is the constant term is not divisible by either the x-coefficient or the y-coefficient?]

Method: Find any point and use the slope.

Example 6: $2x + 3y = 7$. [7 is not divisible by 2 or by 3]

Choose an x-value, substitute, and solve for y, OR choose a y-value, substitute, and solve for x.]

Choosing x=0 or x=1 in this example give fractions for y. Choose x=2.

$2(2) + 3y = 7 \qquad 4 + 3y = 7 \qquad 3y = 3 \qquad y = 1$

Plot the point (in this example, (2,1))

Write the equation in slope-intercept form. ($y = -\frac{2}{3}x + \frac{7}{3}$ in this example)

Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)

From the point go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

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7) Find the slope of the linear function that passes through $(-1,3)$ and $(3,-4)$

step 1: Write the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$\ast$ memorize $\ast$
slope formula
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

step 2: Decide which point you will call (x_2, y_2). Be consistent when substituting!

$$\underset{(x_1,\, y_1)}{(-1,\, 3)} \quad \text{and} \quad \underset{(x_2,\, y_2)}{(3,\, -4)}$$

step 3: Substitute the four values + simplify.

$$m = \frac{-4-3}{3-(-1)}$$

$$\boxed{m = \frac{-7}{4}}$$

8) Find the equation of the line through $(-1,3)$ and $(3,-4)$

step 1: Find slope.
We did this in 7 $m = -\frac{7}{4}$.

step 2: Substitute $m = -\frac{7}{4}$ and one point into the point-slope formula

$$y - y_1 = m(x - x_1)$$

$\ast$Memorize$\ast$
Point slope formula
$y - y_1 = m(x - x_1)$

continued.

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$$y-3=-\frac{7}{4}(x-(-1))$$

step 3: Simplify to slope-intercept form.

$$y-3=-\frac{7}{4}(x+1)$$

$$y-3=-\frac{7}{4}x-\frac{7}{4}$$

$$y=-\frac{7}{4}x-\frac{7}{4}+3\cdot\frac{4}{4}$$ isolate y.

$$y=-\frac{7}{4}x-\frac{7}{4}+\frac{12}{4}$$ find common denom.

$$\boxed{y=-\frac{7}{4}x+\frac{5}{4}}$$ equation of line

$$\boxed{f(x)=-\frac{7}{4}x+\frac{5}{4}}$$ linear function.

9 Find a linear function g such that $g(1)=5$ and $g(5)=17$.
Then find $g(-3)$.

step 1: Change function notation to ordered pairs

$g(1)=5$ means $x=1$, $y=5$ $\Rightarrow (1,5)=(x_1,y_1)$

$g(5)=17$ means $x=5$, $y=17$ $\Rightarrow (5,17)=(x_2,y_2)$

Proceed as before: Find slope

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{17-5}{5-1}=\frac{12}{4}=3$$

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substitute into point-slope formula:

$y - 5 = 3(x - 1)$

$y - 5 = 3x - 3$ distribute

$y = 3x - 3 + 5$ isolate y.

$y = 3x + 2$ combine

$\boxed{g(x) = 3x + 2}$ change to function notation given

Find $g(-3)$.

Substitute $x = -3$

$g(-3) = 3(-3) + 2$

$= -9 + 2$

$= \boxed{-7}$

(10) Write a linear function having slope 0.32 and y-intercept 129.

step 1: Write slope-intercept form.

$f(x) = mx + b$.

Step 2: Substitute

$\boxed{f(x) = .32x + 129}$

Math 60 How to Write the Equation of a Line as a Linear Function

Step 1: Recognize if the line is vertical. Write equation $x = x - coordinate$.

How to know if a line is vertical:

- It says "vertical".
- It says "slope undefined".
- It is parallel to another line with undefined slope (vertical).
- It is parallel to another line whose equation is $x = x - coordinate$ (vertical).
- It is perpendicular to a horizontal line, $y = y - coordinate$.
- It is perpendicular to a horizontal line, slope = 0.
- It is parallel to the y-axis.
- It is perpendicular to the x-axis.

IMPORTANT: Vertical lines are not functions!

Step 2: Recognize if the line is horizontal. Write equation $f(x) = y - coordinate$.

How to know if a line is horizontal?

- It says "horizontal".
- It says "slope 0".
- It is parallel to another line with zero slope (horizontal).
- It is parallel to another line whose equation is $y = y - coordinate$ (horizontal).
- It is perpendicular to a vertical line, $x = x - coordinate$.
- It is perpendicular to a vertical line, slope undefined.
- It is parallel to the x-axis.
- It is perpendicular to the y-axis.

Step 3: Given slope and a point:

If the point is the y-intercept $(0, b)$, substitute into $f(x) = mx + b$.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace y by $f(x)$.

Step 4: Given two points:

Find the slope using the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace y by $f(x)$.

Step 5: Given "parallel to ______" and a point.

Find the slope of the given line by writing in $y = mx + b$

Use that same slope.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace y by $f(x)$.

Step 6: Given "perpendicular to ______" and a point.

Find the slope of the given line by writing in $y = mx + b$

Take the opposite and reciprocal of that slope to get the new slope.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace y by $f(x)$.